

# Damping and frequency shift in the oscillations of two colliding Bose-Einstein condensates

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**Abstract.** We have investigated the center-of-mass oscillations of a  $^{87}\text{Rb}$  Bose-Einstein condensate in an elongated magneto-static trap. We start from a trapped condensate and we transfer part of the atoms to another trapped level, by applying a radio-frequency pulse. The new condensate is produced far from its equilibrium position in the magnetic potential, and periodically collides with the parent condensate. We discuss how both the damping and the frequency shift of the oscillations are affected by the mutual interaction between the two condensates, in a wide range of trapping frequencies. The experimental data are compared with the prediction of a mean-field model.

**PACS.** 03.75.Fi Phase coherent atomic ensembles; quantum condensation phenomena – 05.30.Jp Boson systems – 32.80.Pj Optical cooling of atoms; trapping – 34.20.Cf Interatomic potentials and forces

## 1 Introduction

The issue of the interaction between Bose-Einstein condensates in different internal states has deserved much attention in the recent literature, both from the experimental [1–6] and theoretical point of view [7–9]. The study of these mixtures of quantum fluids includes several important topics, as the characterization of the dynamical behaviour, the response in presence of an external coupling, the phase coherence and the superfluid properties of the system.

Recently we have demonstrated an experimental method for a sensitive investigation of the interaction between two condensates [6, 10, 11], where two condensates are made to collide after periods of spatially separated evolution and quantitative information is extracted from the resulting collective dynamics.

In this article we report on new measures performed on such system in a wide range of trapping frequencies, which allow a systematic investigation of the effects of the collisions between the two condensates.

The experimental setup follows the scheme reported in reference [6]. We use a radio-frequency (rf) pulse to produce two  $^{87}\text{Rb}$  condensates in the states  $|F = 2, m_f = 2\rangle \equiv |2\rangle$  and  $|2, 1\rangle \equiv |1\rangle$ . Due to the different magnetic moments and the effect of gravity, they are trapped in two potentials whose minima are displaced along the vertical  $y$ -axis by a distance much larger than the initial size of each condensate. As a consequence the  $|1\rangle$  condensate, initially created with the same density distribution and at

the same position of the  $|2\rangle$  condensate, undergoes large center-of-mass oscillations. Moreover, the two condensates periodically collide, and this strongly affects the collective excitations of both the condensates [6]. In particular the center-of-mass oscillation of the  $|1\rangle$  condensate is damped and its frequency is shifted upwards, with respect to the non interacting case.

The article is organized as follows. In Section 2, we describe the experimental setup. Then, in Section 3, we present the experimental results, analyzing the effect of mutual interactions on the center-of-mass motion over a wide range of trap frequencies. The data are compared with the predictions of the Gross-Pitaevskii theory for two coupled condensates at zero temperature, according to the model discussed in reference [12]. Finally, in Section 4 we draw the conclusion.

## 2 Experimental setup

We start with a single  $|2\rangle$  condensate containing typically  $1.5 \times 10^5$   $^{87}\text{Rb}$  atoms at a temperature below 130 nK, confined in a 4-coils Ioffe-Pritchard trap elongated along the horizontal  $z$ -symmetry axis [13], which produces an axially symmetric harmonic potential. The harmonic oscillator axial frequency is held fixed to  $\nu_{z2} = 12.6$  Hz, while the radial frequency  $\nu_{\perp 2}$  has been varied in the range 113–310 Hz by changing the minimum magnetic field  $B_0$  according to the relation  $\nu_{\perp 2} = 221 \text{ Hz}/B_0[\text{G}]^{1/2}$ . The density distribution is an inverted parabola of the Thomas-Fermi (TF) regime [14], and the typical radial and longitudinal radii of the condensate are  $R_{\perp} = 4.3\text{--}2.4 \mu\text{m}$  and

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$R_z = 39\text{--}58 \mu\text{m}$ , in the range of frequencies considered here.

To populate the  $|1\rangle$  state we apply a rf oscillating magnetic field. After the rf pulse the initial  $|2\rangle$  condensate is put into a coherent superposition of different Zeeman  $|m_f\rangle$  sub-levels of the  $F = 2$  state, which then move apart:  $|2\rangle$  and  $|1\rangle$  are low-field seeking states and stay trapped,  $|0\rangle$  is untrapped and falls freely under gravity, while  $|-1\rangle$  and  $|-2\rangle$  are high-field seeking states repelled from the trap.

By fixing the duration and varying the amplitude of the rf field, we control the relative population in different Zeeman sub-levels [15]. In the experiments described here we use a 10 cycles rf pulse at 1.24 MHz with an amplitude  $B = 20$  mG to quickly transfer  $\sim 13\%$  of the atoms to the  $|1\rangle$  state without populating the  $|0\rangle$ ,  $|-1\rangle$  and  $|-2\rangle$  states.

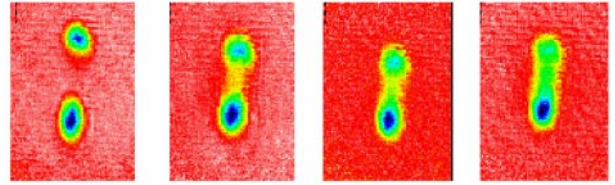
In the frequency domain, the pulse width exceeds by almost one order of magnitude the transition broadening due to the inhomogeneous magnetic field across the condensate. As a result, the coupling strength does not depend on position and  $|1\rangle$  is produced with the same spatial density profile as  $|2\rangle$ . Furthermore, since the characteristic time scale of the collective dynamics is of the order of 1 ms, no evolution occurs for the wavefunctions during the pulse, but, immediately after, excitations start in both the condensates.

The  $|1\rangle$  condensate experiences a trapping potential with lower axial and radial frequencies ( $\nu_1 = \nu_2/\sqrt{2}$ ), whose minimum is displaced along the vertical  $y$ -axis by a distance  $y_0 = g/\omega_{1,2}^2$  due to gravity (“gravitational sagging”). In the frequency range considered here  $y_0$  varies between 19 and  $2.6 \mu\text{m}$ . After the rf-pulse, the  $|1\rangle$  condensate moves apart from  $|2\rangle$ , and begins to oscillate around its equilibrium position. Due to the mutual repulsive interaction, the latter starts oscillating too, though with a much smaller amplitude. Actually, each condensate moves in an effective potential which is the sum of the external potential (magnetic and gravitational) and the mean-field one. Here we have studied the dynamics of the  $|1\rangle$  condensate by varying the permanence time in the trap up to 40 ms.

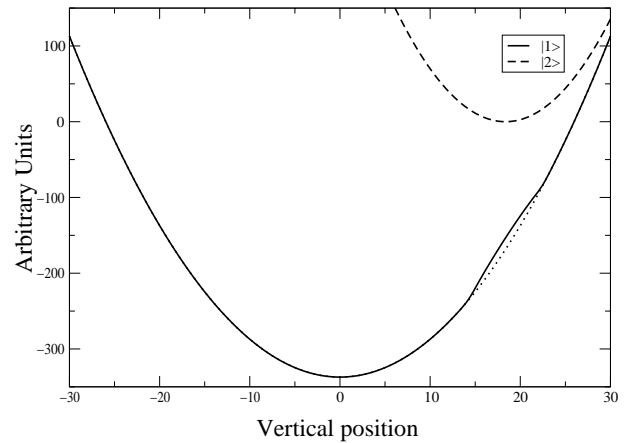
After the release from the magnetic trap, we let the clouds drop and expand for 30 ms, and finally we take an absorption image on a CCD array. We notice that the process of switching off the magnetic field is not instantaneous, and lasts about 1 ms. Moreover it produces a magnetic gradient which affects the initial velocity of the two condensates. The acquired velocities experimentally observed are  $v_{1y} = 0.7 \pm 0.1$  cm/s and  $v_{2y} = 1.4 \pm 0.1$  cm/s, respectively for the  $|1\rangle$  and  $|2\rangle$  condensates. In the following we use an unknown, effective, switch-off delay time as the only fitting parameter for our theoretical model.

### 3 Center-of-mass oscillations

Let us recall the basic features of our system, as reported for the original experiment in reference [6]. The initial configuration corresponds to the stationary ground-state with all the  $N$  trapped atoms in the  $|2\rangle$  condensate. Afterwards,



**Fig. 1.** Images of the two condensates before and after the collision (from left to right).



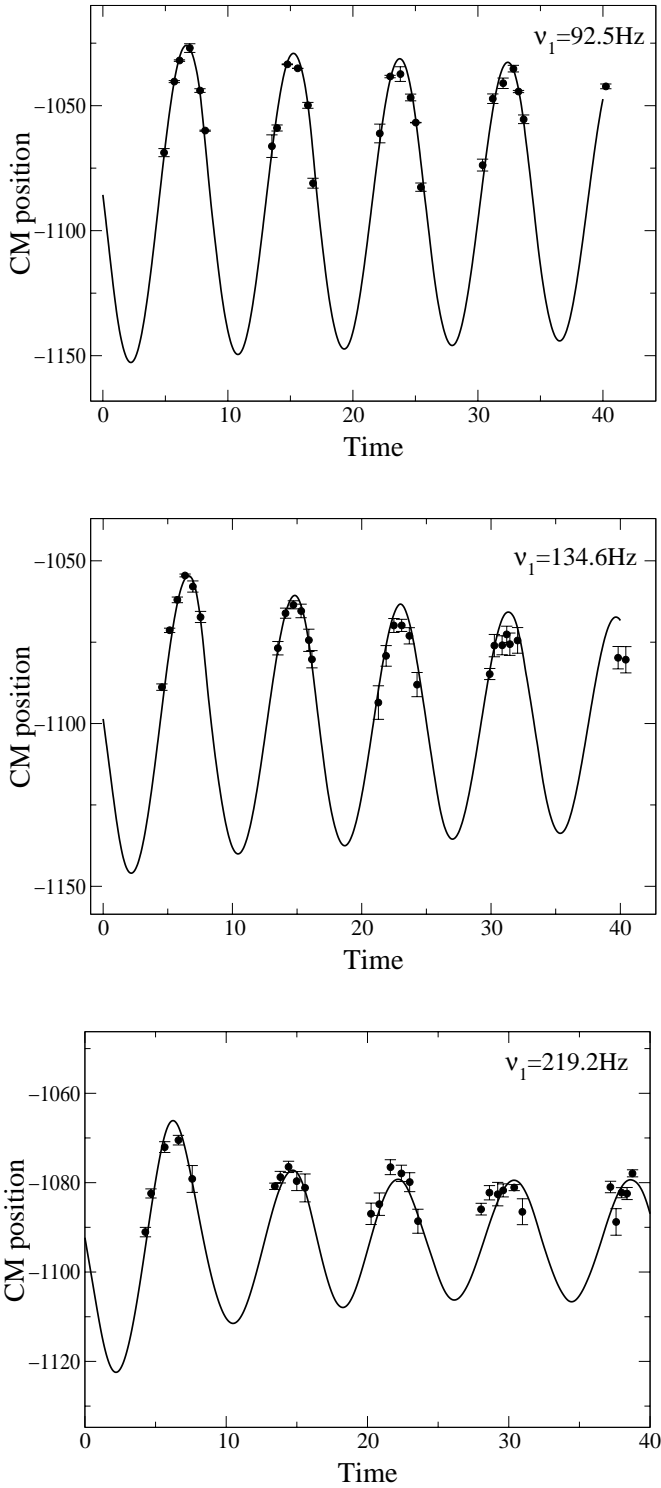
**Fig. 2.** Trapping potentials for the two condensates in the vertical direction for  $\nu_{\perp 2} = 116$  Hz (lengths are in units of  $a_{\perp 2} = [\hbar/(m\omega_{\perp 2})]^{1/2} \simeq 1.00 \mu\text{m}$ ). The  $|1\rangle$  condensate moves in an effective potential (solid line) which is modified by the mean field repulsion of  $|2\rangle$  (right side).

at  $t = 0$ ,  $N_1 \simeq 0.13N$  atoms are transferred from the  $|2\rangle$  to the  $|1\rangle$  state, the former remaining with  $N_2 = N - N_1$  atoms.

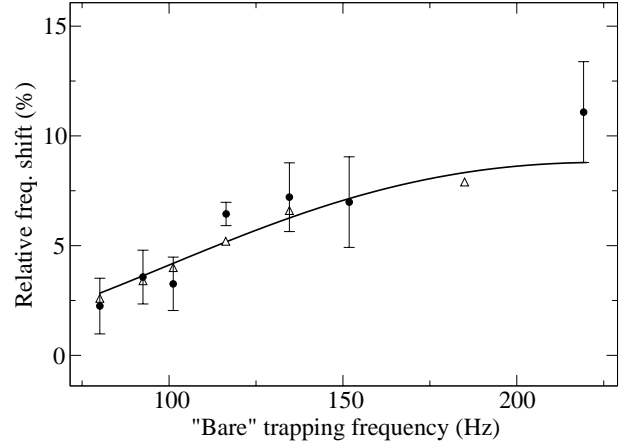
The dynamics of the condensate  $|2\rangle$  is characterized by small amplitude center-of-mass oscillations and by shape oscillations which are both affected by the mutual repulsion with the other condensate when they periodically collide (see Fig. 1) [6].

Here we are mainly interested in the motion of the  $|1\rangle$  condensate, which is on the contrary dominated by large oscillations, whose frequency  $\nu_1$  turns out to be larger than the “bare” trapping frequency  $\nu_{\perp 1}$ . In fact the condensate  $|1\rangle$  moves in an anharmonic effective potential (see Fig. 2) due to the presence of the condensate  $|2\rangle$ , which provides an extra repulsion with a consequent up-shift of frequency. This shift is one of the clear signatures of the mutual interaction, since it only occurs if the two condensates overlap periodically.

Furthermore, the oscillations appear to be damped. At the level of the Gross-Pitaevskii theory this damping is not due to dissipative processes (the total energy is conserved), but to a redistribution of the kinetic energy initially associated with the center-of-mass motion of the  $|1\rangle$  condensate, which is eventually shared among the other degrees of freedom of the system (center-of-mass of the other condensate and internal excited states of both of them) [12].



**Fig. 3.** Center-of-mass oscillations of the  $|1\rangle$  condensates after the ballistic expansion for three values of the “bare” trapping frequency ( $\nu_{\perp 1} = 92.1, 134.6, 219.2$  Hz, from top to bottom). The measured center-of-mass positions (arbitrary units) are plotted as a function of the trapped evolution time (in units of  $\omega_{\perp 1}^{-1}$ ), and compared with the prediction of our 2D model (solid line).



**Fig. 4.** Relative shift of the  $|1\rangle$  oscillation frequency *versus* its “bare” trapping frequency, *i.e.* in absence of interactions with  $|2\rangle$ . The experimental data are compared with the prediction of the classical model (solid line) in reference [12] and of the 2D-GP model (empty triangles).

To investigate the role of the interactions in the large center-of-mass oscillations of the  $|1\rangle$  condensate in different dynamical regimes, we have carried out a systematic analysis by varying the trap frequency  $\nu_{\perp 1}$  in the range 80–220 Hz (which corresponds to the above mentioned range 113–310 Hz for  $\nu_{\perp 2}$ ).

In Figure 3 we show the center-of-mass motion of the  $|1\rangle$  condensate, for some values of the radial trapping frequency, by comparing the experimental data with the prediction of the 2D Gross-Pitaevskii model discussed in references [6,12]. In this case the only fitting parameter in the theoretical curves is the delay time caused by the non instantaneous switch-off of the trapping potential. Its value is  $\simeq 0.7$  ms. We notice also that in our model the effect of the expansion on the center-of-mass motion is considered as a simple free fall under gravity, neglecting any residual interaction between the condensates. Actually, although the two clouds acquire different velocities during the release from the trap, and cross each other in the fall, we expect them to quickly become so dilute, due to the fast radial expansion, that we can neglect their mutual mean-field repulsion. This is also supported by numerical simulation including interaction in the early stages of expansion [16].

The values for the frequency shift and the damping time in the whole range of frequency are summarized in Figures 4 and 5 respectively. In Figure 4 the relative shift of the  $|1\rangle$  oscillation frequency is plotted *versus* its “bare” trapping frequency, *i.e.* in absence of interactions with  $|2\rangle$ . The experimental data are compared with the prediction of the classical model (solid line) and of the 2D-GP model (triangles). In Figure 5, we show the “quality factor”  $Q = \omega_{\perp 1} \tau$  of the oscillator as a function of  $\nu_{\perp 1}$ ,  $\tau$  being the damping time, comparing experimental points and theoretical predictions for our 2D model.

We notice that, concerning the frequency shift, there is a general agreement (within the experimental uncertainty) between GP theory and experiment in the whole

range of frequency, except for the frequency of the original experiment (116.3 Hz). By the way, we have found that in this frequency range around 116.3 Hz there is an almost degeneracy between the energies of the two condensates (mean field + kinetic + potential energy that the two condensates can exchange during the collisions, see Ref. [12]). At this stage, we cannot tell whether this can play some physical role, or it is a pure coincidence.

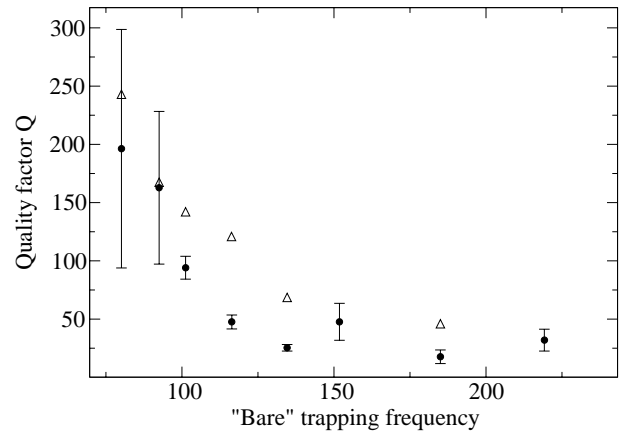
On the contrary the observed damping is generally underestimated by the 2D GP model. This is not surprising, since there could be other sources of damping not included in our mean-field description. To be more specific, one should consider the following points.

(i) The theoretical curves are based on a 2D model which only includes the radial degrees of freedom in the  $xy$ -plane. In fact, being the geometry of our condensates strongly elongated in the longitudinal direction, we have approximated the system as a uniform cylinder, by freezing out the axial dynamics (see Ref. [12] for a comprehensive discussion). Therefore, the remaining contribution to the damping could in part arise from the excitations of axial modes (possible distortions of the shape of the two condensates along  $z$ ), which are ignored in the present analysis. Anyway, there are indications from preliminary 3D simulations that these contributions should not be so important, at least for the case at 116.3 Hz.

(ii) We observe a certain fragmentation of the  $|1\rangle$  condensate after some collision. This may have an influence on the measured center-of-mass since the density distribution develops low density asymmetric wings, which we might be unable to detect properly.

(iii) From the analysis of the experimental data there are indications in support of the occurrence of atom (and energy) losses from both condensates. In fact, the relative number of atoms for the  $|1\rangle$  state ( $N_1/N$ ) is in average a decreasing function of the trapping time. We remind that, although each experimental point corresponds to a different experiment (each point corresponds to a condensates released from the trap after a variable time), they are all performed with almost the “same” initial conditions. In principle we can exclude that these losses originate from the condensate finite lifetime (0.7(1) s) or heating ( $dT/dt = 0.11(2) \mu\text{K/s}$ ), due to the short permanence times in the trap (less than 40 ms). Actually, this behaviour could be explained with a loss of atoms due to binary collisions between atoms of the two condensates, an effect that is not included in our GP mean-field theory.

Concerning this last point, an appealing mechanism could be the occurrence of elastic scattering processes between atoms of the two overlapping condensates, as discussed in reference [17] for the impurities motion inside a Bose condensate [18] and in the analysis of matter 4-waves mixing [19]. In fact, since Bose-Einstein condensates are superfluids, when the relative velocity of the two condensates exceeds a critical value of the order of the sound velocity, we could expect the GP mean-field approximation to break down and “elastic scattering” losses to take place. Indeed, in our experiment, the two condensates collide at a velocity which is larger than the (maximum)



**Fig. 5.** Measured “quality factor”  $Q \equiv \omega_{\perp 1} \tau$  for the  $|1\rangle$  oscillation versus its “bare” frequency compared with the prediction of the 2D-GP model (empty triangles).

speed of sound at the top of  $|2\rangle$ ,  $c_0 = \hbar \sqrt{4\pi n_2 a_{22}} / m$ , by a factor  $\alpha$  that can be tuned by changing the radial frequency. In the frequency range considered,  $\alpha$  goes from 4.2 at  $\nu_{\perp 2} = 113$  Hz to 2.1 at  $\nu_{\perp 2} = 310$  Hz. So, we should expect the effect of the “elastic scattering”, if any, to be more pronounced at low radial frequency. As a consequence the results of the GP model should be more accurate at high frequency. This conclusion, however, is not supported by the experimental observations.

A detailed theoretical analysis of our experiment in this respect is under way at the NIST group, in Gaithersburg (USA). Preliminary results give a better agreement for the damping time with the inclusion of elastic scattering [16], at least for the case of the original experiment in reference [12].

## 4 Conclusions

To summarize, we have analyzed the large center-of-mass oscillations of a Bose-Einstein condensates in a strongly elongated magnetic trap, over a wide range of frequencies. The condensate is produced by transferring part of the atoms from a parent condensate in the  $|F = 2, m_f = 2\rangle$  state to the  $|F = 2, m_f = 1\rangle$  state. The new condensate is created far from its equilibrium position in the magnetic potential, and undergoes large oscillations, periodically colliding with the parent condensate. We have shown that the mutual interactions between the two condensates deeply affect both the amplitude and the frequency of the oscillations. The experimental data are compared with the prediction of a 2D model based on the Gross-Pitaevskii theory for two coupled condensates at zero temperature. This model accounts for the basic features of the system, and the agreement with the data is generally satisfactory. Nevertheless, the discrepancy observed for the damping (and in some case also for the frequency) suggests that processes of atom losses (*e.g.* elastic scattering losses [17]) could play a significant role beyond the mean-field theory.

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